The Illuminated Scores

The Architectural Design of Musical Form

Orlay Alonso - Author

A special thanks to Dr. Orlando Alonso,
Dr. Charles Atkinson and Emily Alonso-Taub for
their guidance and support.

The Illuminated Scores are designed as scholarly editions for performing artists and educators, to portray the form and structure of a musical composition. They incorporate semiotic tools into the study of musical language intended to make the architectural design visible in a coherent format. The purpose of the study is to develop a method of representing music graphically that differs from the established score layout, reorganizing content in a manner that allows one to overcome the constraints under which musical skills are developed in the current model of publishing music.

The project has developed into a new publishing format containing the author's analysis of works by Bach, Mozart and Schoenberg. In addition, diagrams and color-coding illustrate how mathematics and music are combined to explain the concept of balance in musical form, thereby revealing the inherent logic of a composer's cohesive thought process.

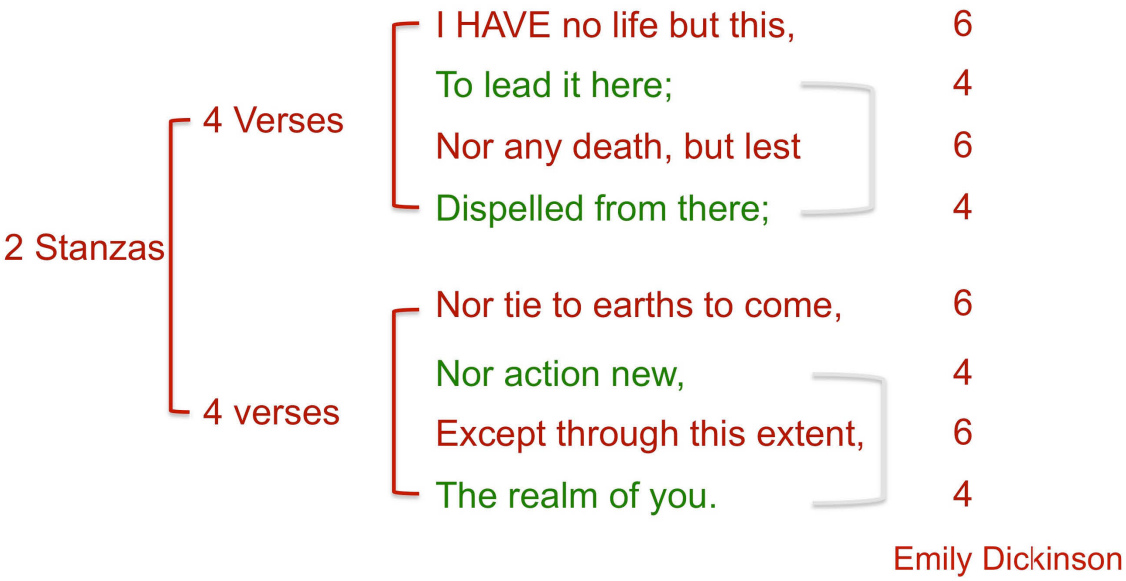
The following is a transcription of a poem by Emily Dickinson in the layout of prose rather than verse in order to show a parallel comparison of what the author hopes to accomplish musically:

"I have no life but this, to lead it here; nor any death, but lest dispelled from there; nor tie to earths to come, nor action new, except through this extent, the realm of you."

- Emily Dickinson

Immediately, one can deduce that it is very difficult to analyze a poem in prose. Such format is not very helpful to the reader in the poetic delivery of the work. The advantages of looking at the poem in its rightful shape or originally intended design can be examined in example 1 below. One can clearly see the intended "Architectural Design" which is comprised of one poem made up of two stanzas with four verses per stanza, where green represents rhyme as in a Ballad alternating six and four syllables per verse.

Example 1:



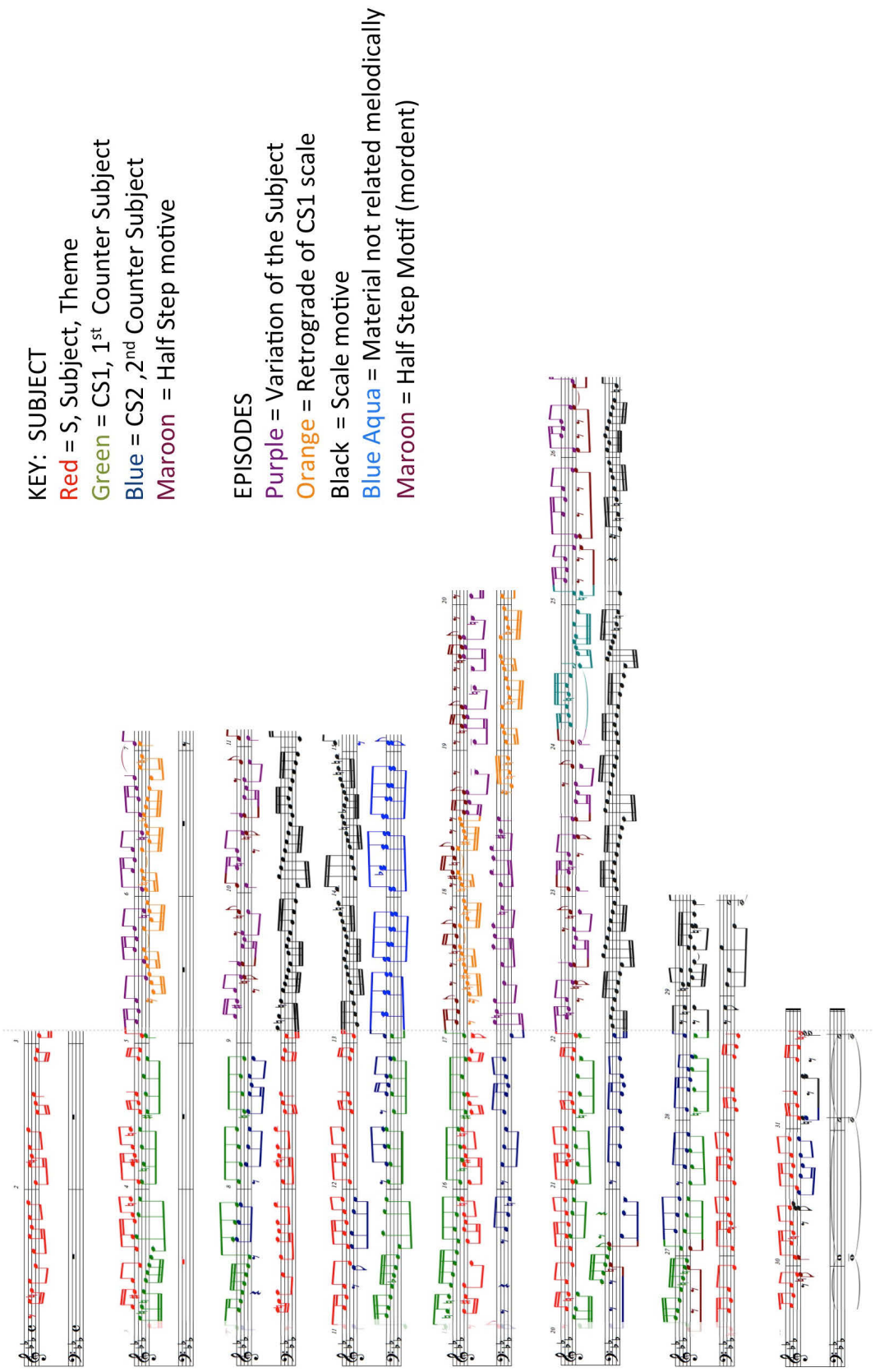
After analyzing the poem in both formats, one can begin to realize that music has been published in "prose" for hundreds of years, written with the economy of space in mind from a printing perspective. It is the 21st Century and one no longer needs to worry about what the size of the score is, and in this digital era sheet music is more likely to be in a virtual score format. This realization led the author to the current project.

The Illuminated Scores consist of a new platform or score layout that portrays the form and structure of any given musical composition in a format similar to that of poetic verse, rather than that of continuous prose, where one measure follows another in no relevant way. Phrase lengths, melodic relationships, harmonic structure, rhythmic patterns, and the number of measures in a system will play a significant role in the visual layout of the work. The motivation for the creation of such editions grew out of the need to enable performers and educators to experience an immediate representation of the overall formal design of a musical composition in order to attain a deeper understanding of the meaning of music and form, from the first moment one lays eyes on the score.

Once music is organized in a coherent phrase structure, the form reveals itself. It is the author's hypothesis that this new perspective will impact the ways we learn and teach musical form and its structure.

Example 2 represents the proposed score layout of a well-known fugue by J. S. Bach which consists of organizing the score on the page in complete musical thoughts commonly referred to as phrases.

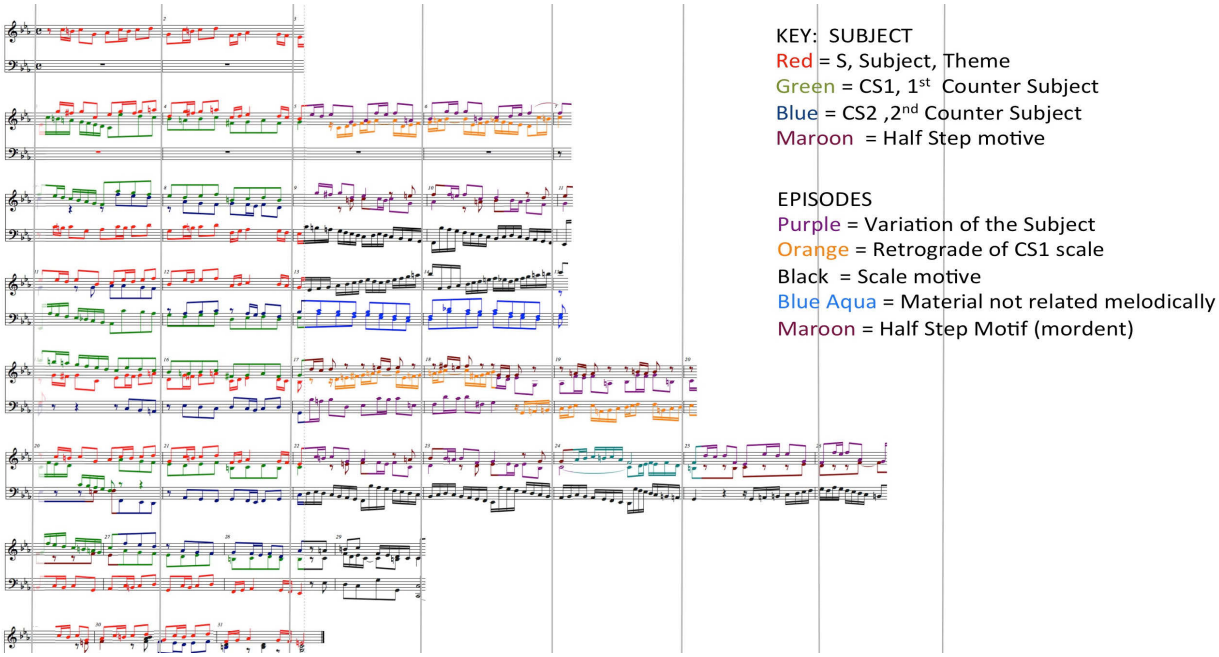
Example 2: Fugue No. 2 in C minor, BWV 847



The parameters of the method are as follows:

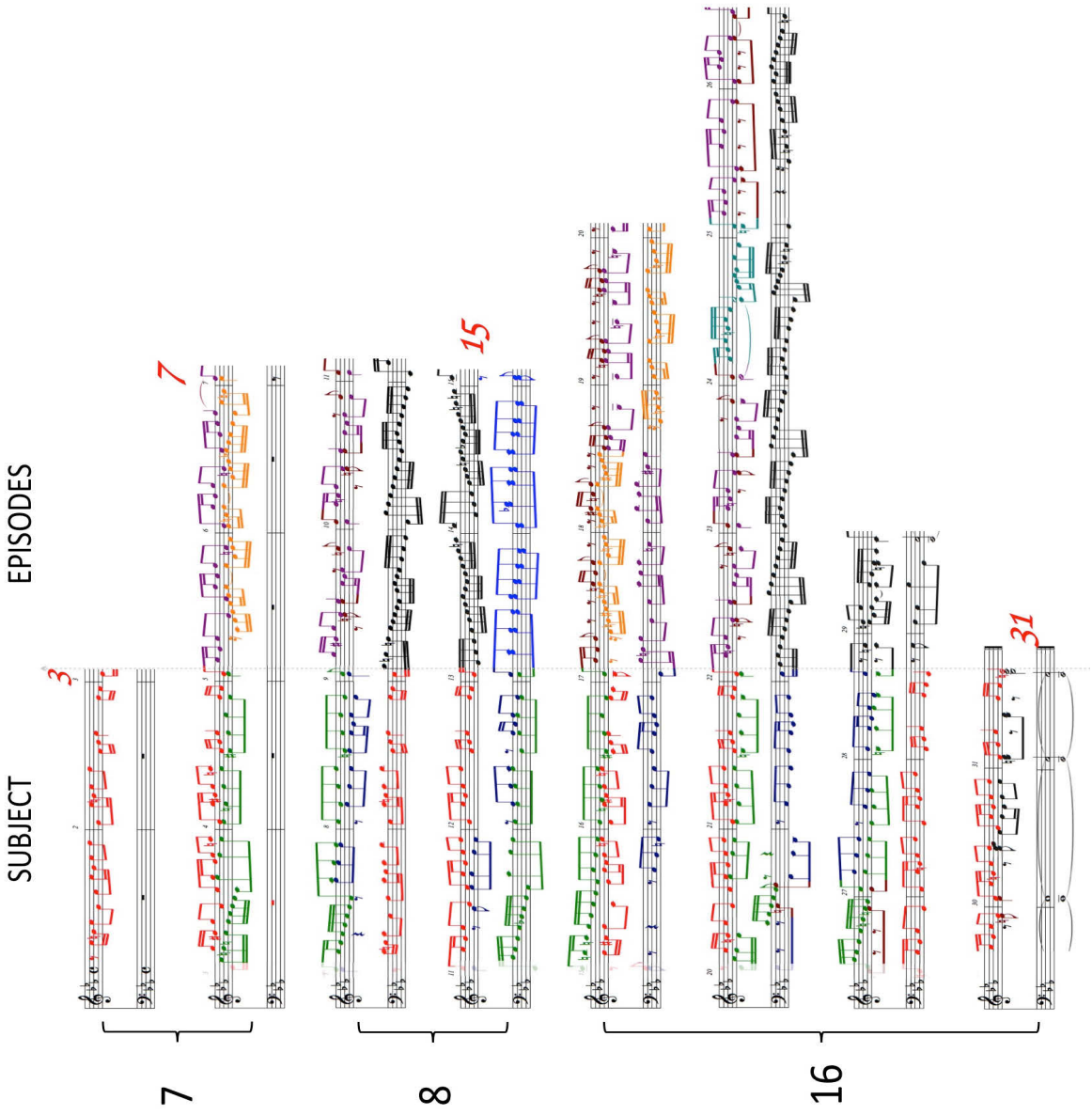
- All measures are equally divided through space (please keep in mind that music is an architectural structure that takes place through time and is audible only through time)
- All measures are of equal length from bar line to bar line in the same time signature (notice that the lines are perfectly aligned and equidistant in example 3)
- All notes should be notated with an even beat spacing (must correspond with the time it takes to play)
- All bar lines should create a straight line down the page as to create a perfect uniform visual experience
- The number of measures in a system is dictated by the length of the phrase
- Phrase lengths are to be notated as they are regardless of where they fall within the bar lines, and should be written as though the bar lines don't exist
- Colors connect similar or equal melodic material only, yet colors can also be used to show related harmonic and rhythmic patterns.

Example 3:



After following the procedures of this new approach, one begins to notice that mathematical proportions come to light. In example 4 on the left side of the score in black are the mathematical proportions and on the right side in red the measure numbers that book end each phrase. The work is 31 measures long with proportions of 8 and 16 (doubling in size) and proportions of 3, 7, 15 and 31 in red. Could Bach have based the formal architectural structure of this fugue on the Mersenne numbers?

Example 4:



In examples 5 and 6 while looking in more detail at the mathematical aspects of form in this work, we find the Mersenne primes shown in red. Please note that even though 15 is not a prime number it is nonetheless an integral part of the Mersenne numbers' sequence.

Example 5:

Prime numbers up to 31: 2 3 7 11 13 17 19 23 29 31

Mersenne Prime Sequence –

$M_p = 2^p - 1$; *whenever* M_p *prime,* p *– prime*

$1 + 2 = 3 \quad 2^2 - 1$

$1 + 2 + 4 = 7 \quad 2^3 - 1$ 3 + 4 = 7

$1 + 2 + 4 + 8 = 15 \quad 2^4 - 1$ 7 + 8 = 15

$1 + 2 + 4 + 8 + 16 = 31 \quad 2^5 - 1$ 15 + 16 = 31

Example 6:

- Mersenne Numbers are numbers of the form: $M_n = 2^n - 1,\ n \geq 0$
- A geometric sequence is a sequence where the quotient of to consecutive terms is a constant, which is called the ratio of the geometric sequence.
- A geometric series is a series (sequence of sums) whose terms (addends) form a geometric sequence.
- A Mersenne number is a term of the geometric series of ratio 2.

Number of iterations (n)	Geometric sequence of ratio 2	Finite sums of the sequence	Sums (Mersenne Numbers)
0	$2^0 = 1$	1	1
1	$2^1 = 2$	$1+2$	3
2	$2^2 = 4$	$1+2+4$	7
3	$2^3 = 8$	$1+2+4+8$	15
4	$2^4 = 16$	$1+2+4+8+16$	31
. . .	. . .	. . .	. . .
$n-1$	2^{n-1}	$1+2^1+2^2+\cdots+2^{n-1}$	$2^n - 1$

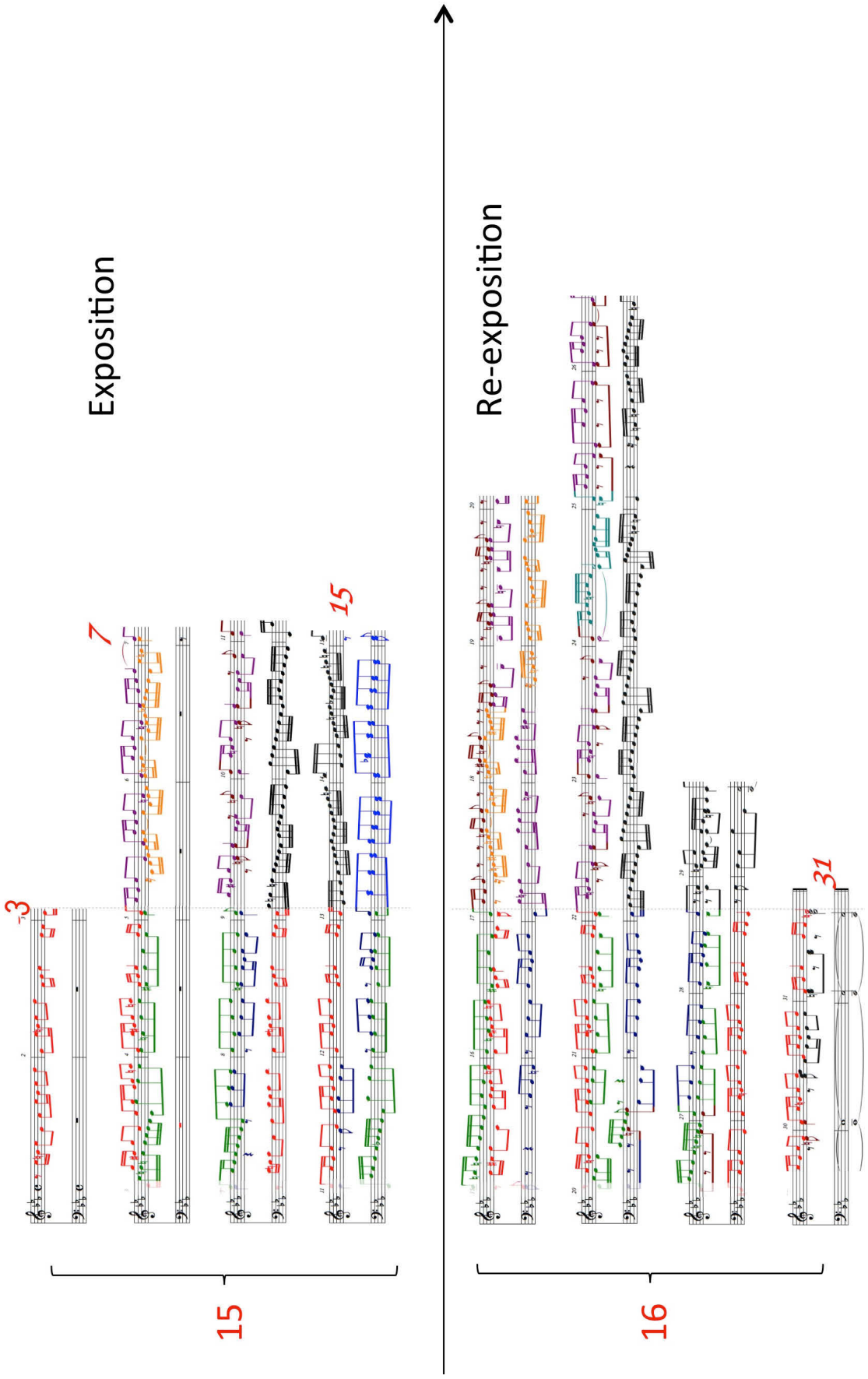
- A prime number (building blocks of numbers) is a number greater than 1 that is divisible only by 1 and by itself.
- Prime numbers up to 31: 2 3 7 11 13 17 19 23 29 31
- A Mersenne prime is Mersenne number which is a prime number. The first four Mersenne primes are 3, 7, 31 and 127

The author speculates that it is not only plausible but appropriate to correlate the work of Bach with Mersenne, based on this fugue's proportions and on the observation that not only was Mersenne a mathematician, instrument builder and theorist but his contributions to equal-temperament tuning could tie him directly to Bach's Well-Tempered Clavier.

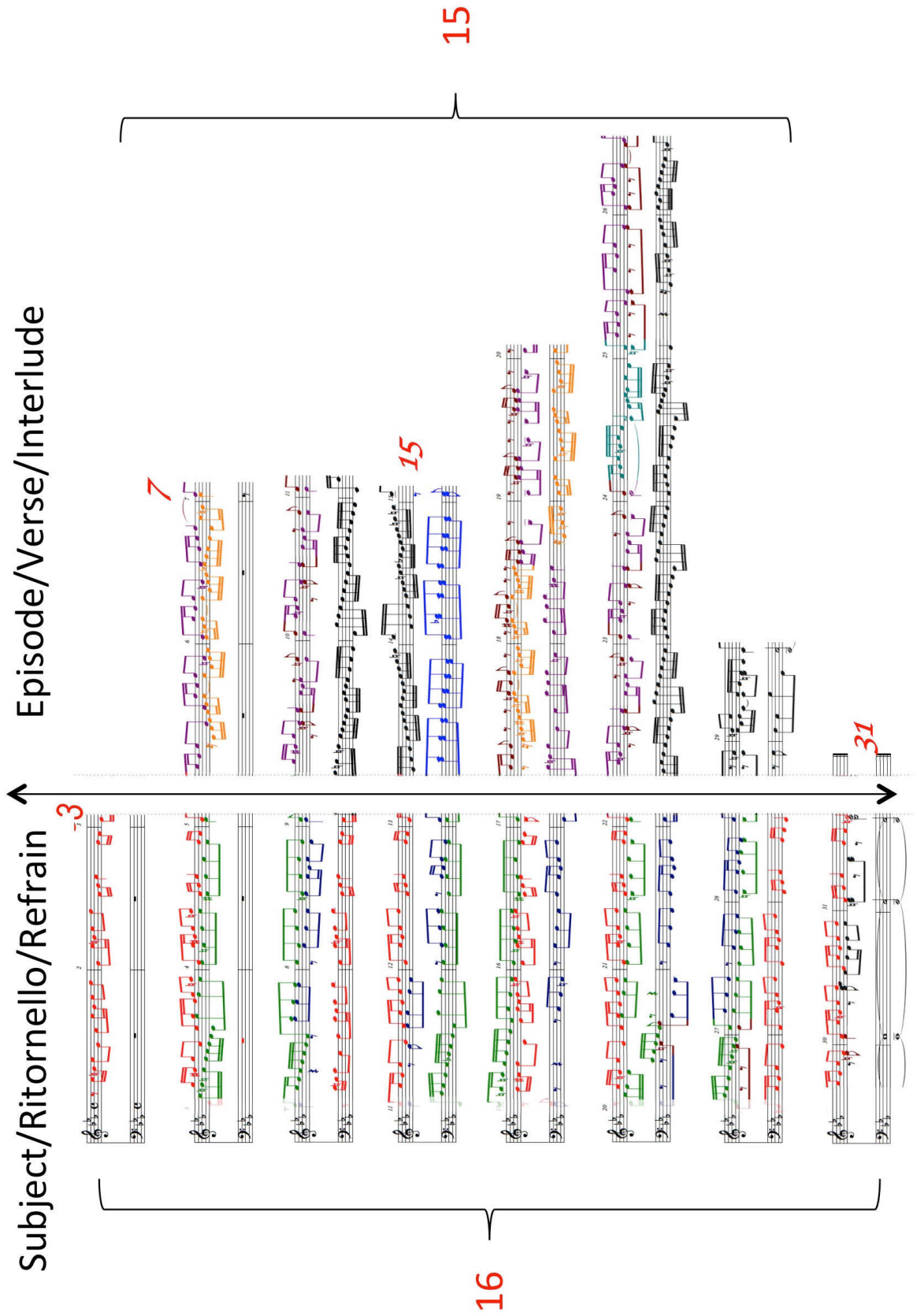
After having reviewed the mathematical terminology one can apply such knowledge to the score. When one "formally" divides the fugue in sections horizontally, into exposition and re-exposition (see example 7), one notices that the exposition is 15 measures long and the re-exposition is 16 measures long. Mathematically speaking one would be more satisfied with an equal division of 16 and 16, adding up to the enjoyable 32 which in music is so favored and easily remembered due to the subdivision of rhythm into whole, half, quarter, eighth, sixteenth, thirty-second, etc. Now the mastery of Bach's mathematical knowledge is represented in the connection between examples 7 and 8.

The beauty of the layout lies in the reciprocity of form; it works horizontally as well as vertically thus we find Bach utilizing form as he does harmony and melody in counterpoint at a completely new level. When we divide vertically (example 8) between subject entrances and episodes, one notices that the entrances of the subject, which now act as a ritornello or refrain, are 16 measures long and the episodic verses are 15 measures long.

Example 7:



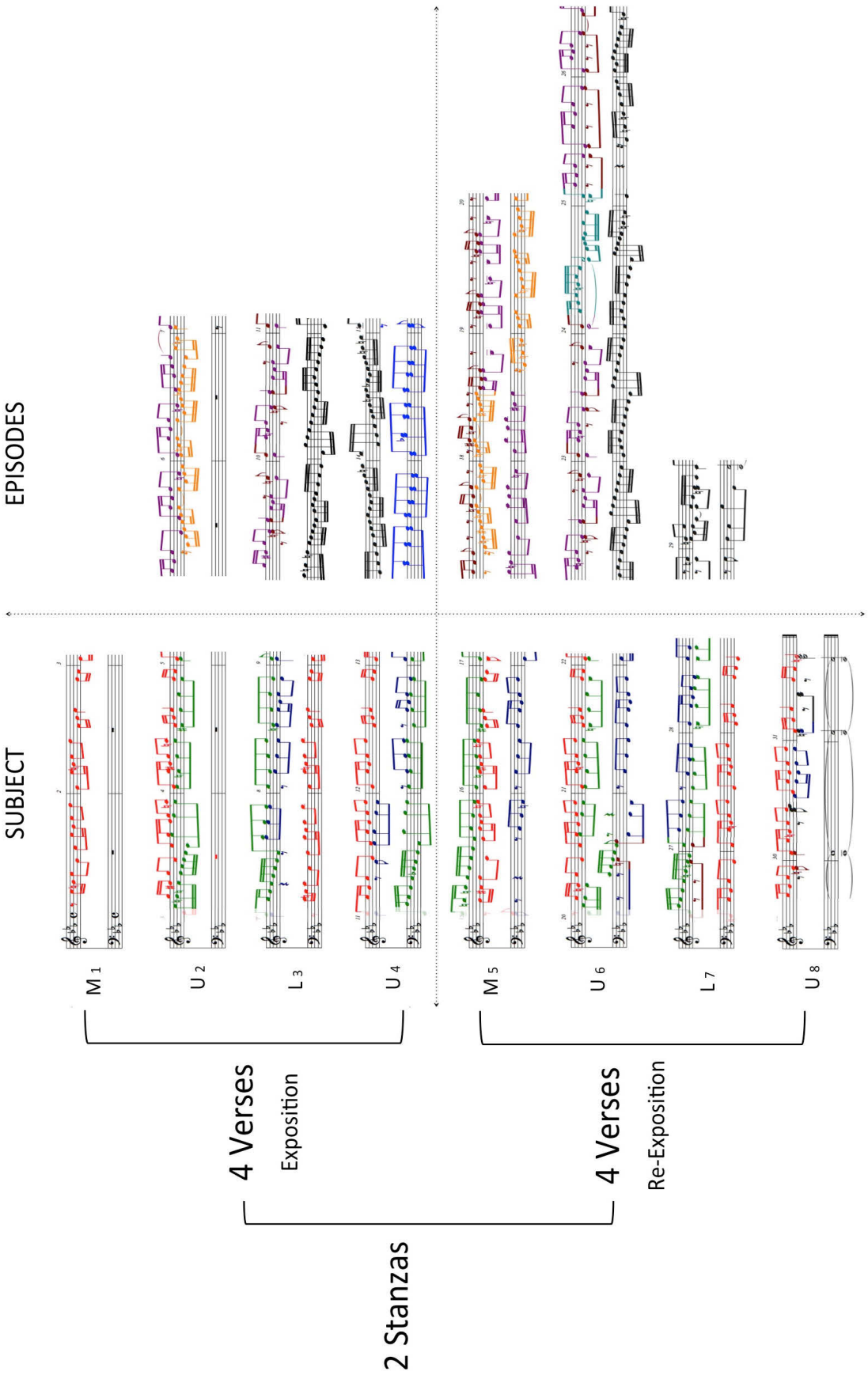
Example 8:



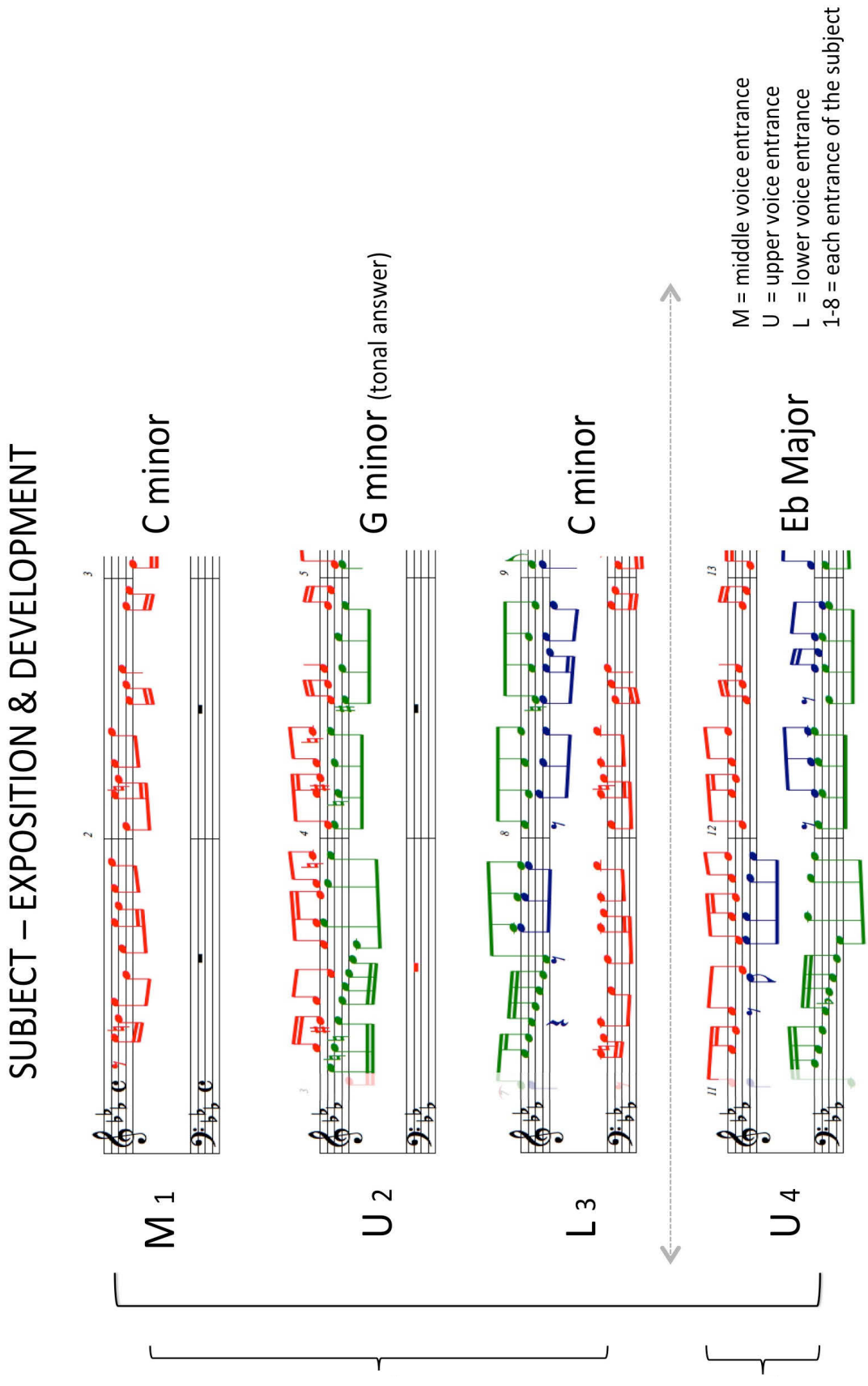
The relevance of this new analytical approach, while preparing for a performance of this work, lies in the ability to phrase related melodies equally to each other, independently of all others, thus giving the performance a depth and variety of musical colors, dynamics and articulation by establishing a clear thematic hierarchy. In other words, each color in this fugue should have its own personal identity, phrasing, articulation, shape and direction; however not necessarily in the same dynamic range. Rarely do we hear performances that depict the rich variety of techniques utilized by Bach.

The author proposes orchestrating each color as the same instrument (or group of instruments) and different colors as different instruments (or stops if using an organ); or on the piano by changing character between subject entrances and episodes, polarizing the sections. When deciding what phrasing to choose the author recommends making sure it is possible to maintain the same identical phrasing and articulation throughout the piece for each color, especially in intricate passages. Most of us commonly settle for knowing where the entrances of the subject are and that by no means is enough to do justice to these miniature masterpieces.

Example 9:

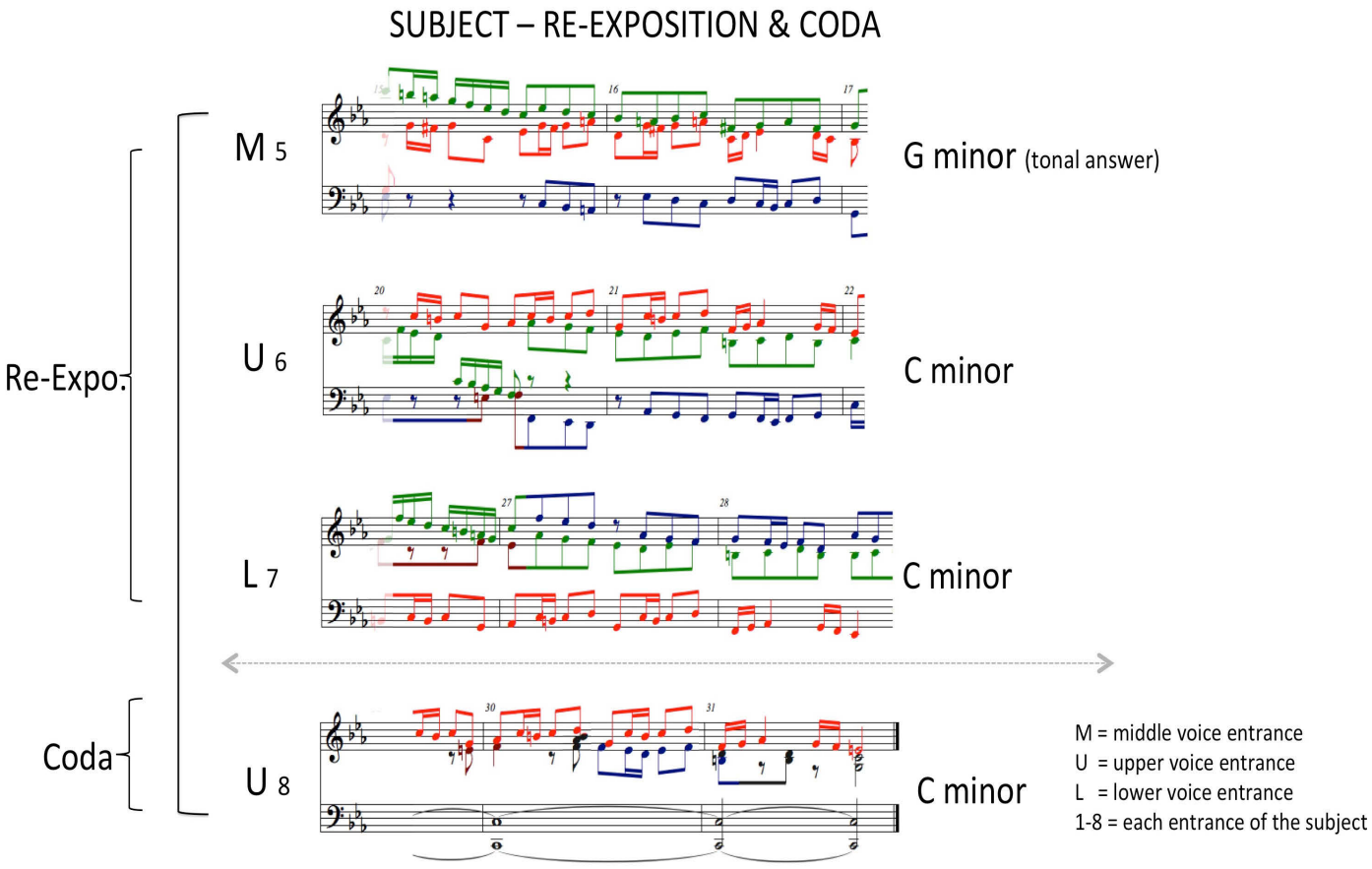


Example 10:

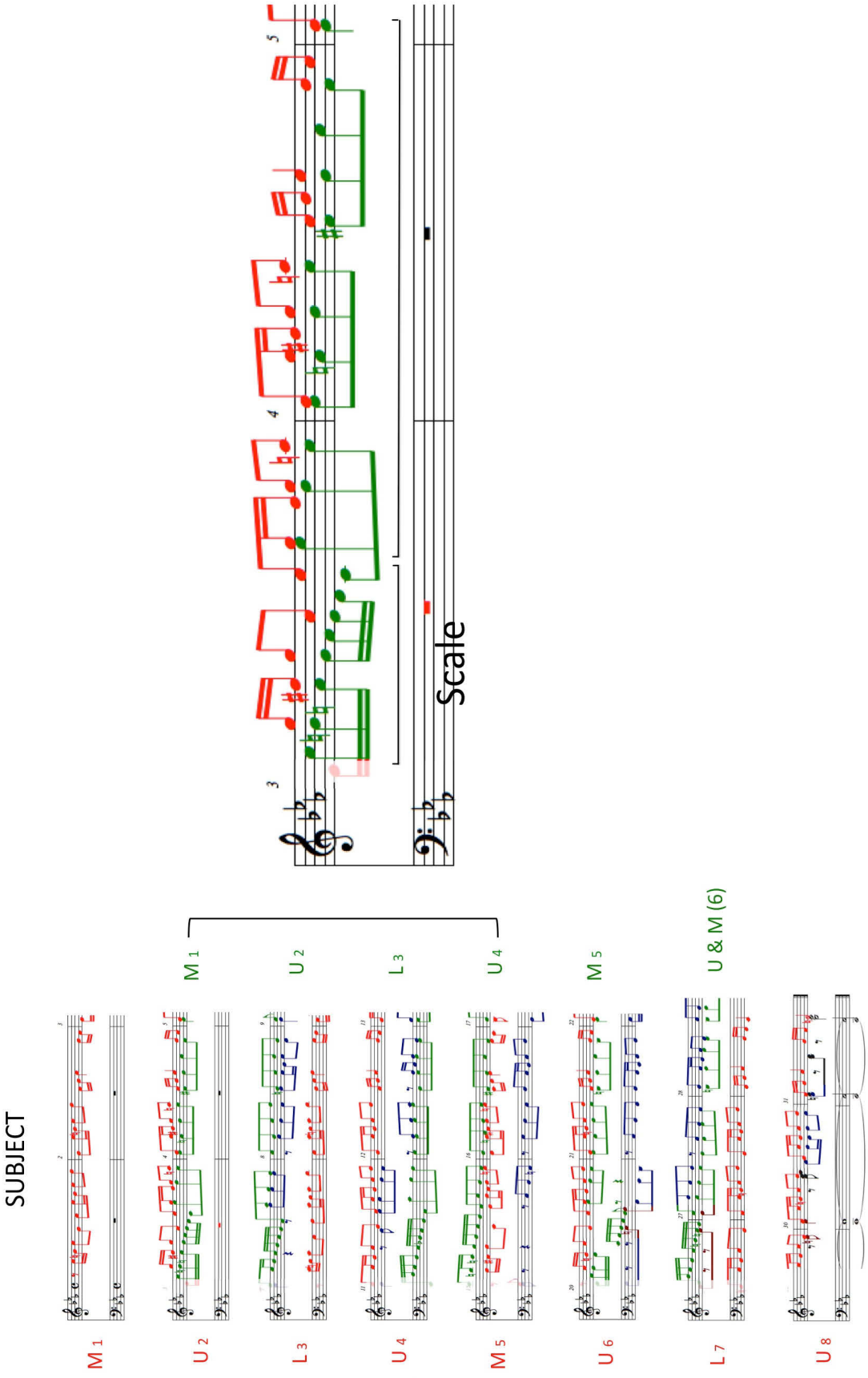


By looking at the exposition by itself (example 10) one can see that the first three entrances of the subject represent the thematic exposition of the subject and its two complimentary counter-subjects (green and blue), and the 4th statement represents the development in the key of E-flat. The author has labeled each system with numbers 1 through 8 and letters MULU (Middle = alto entrance of the subject, Upper = soprano, Lower = tenor) and in example 11 the same pattern is taken up identically in the re-exposition (MULU), however now the last statement U8 represents the coda.

Example 11:

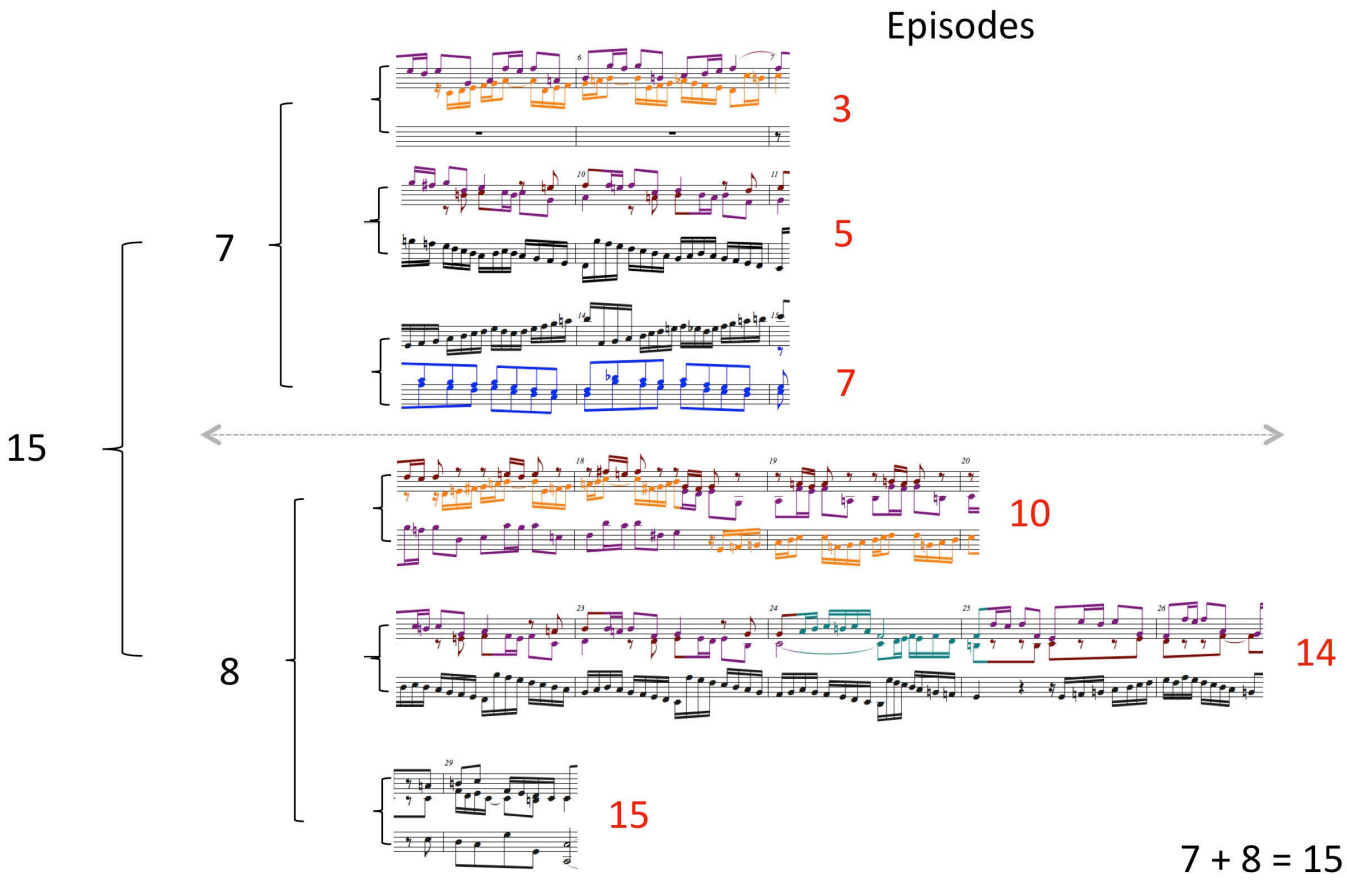


Example 12:



The first counter subject represented here in green (example 12) is mainly constructed of stepwise motion to counteract or balance the angular nature of the subject. Counter subject 1 also mimics or canonizes the pattern (MULU) of the ordered entrances of the subject.

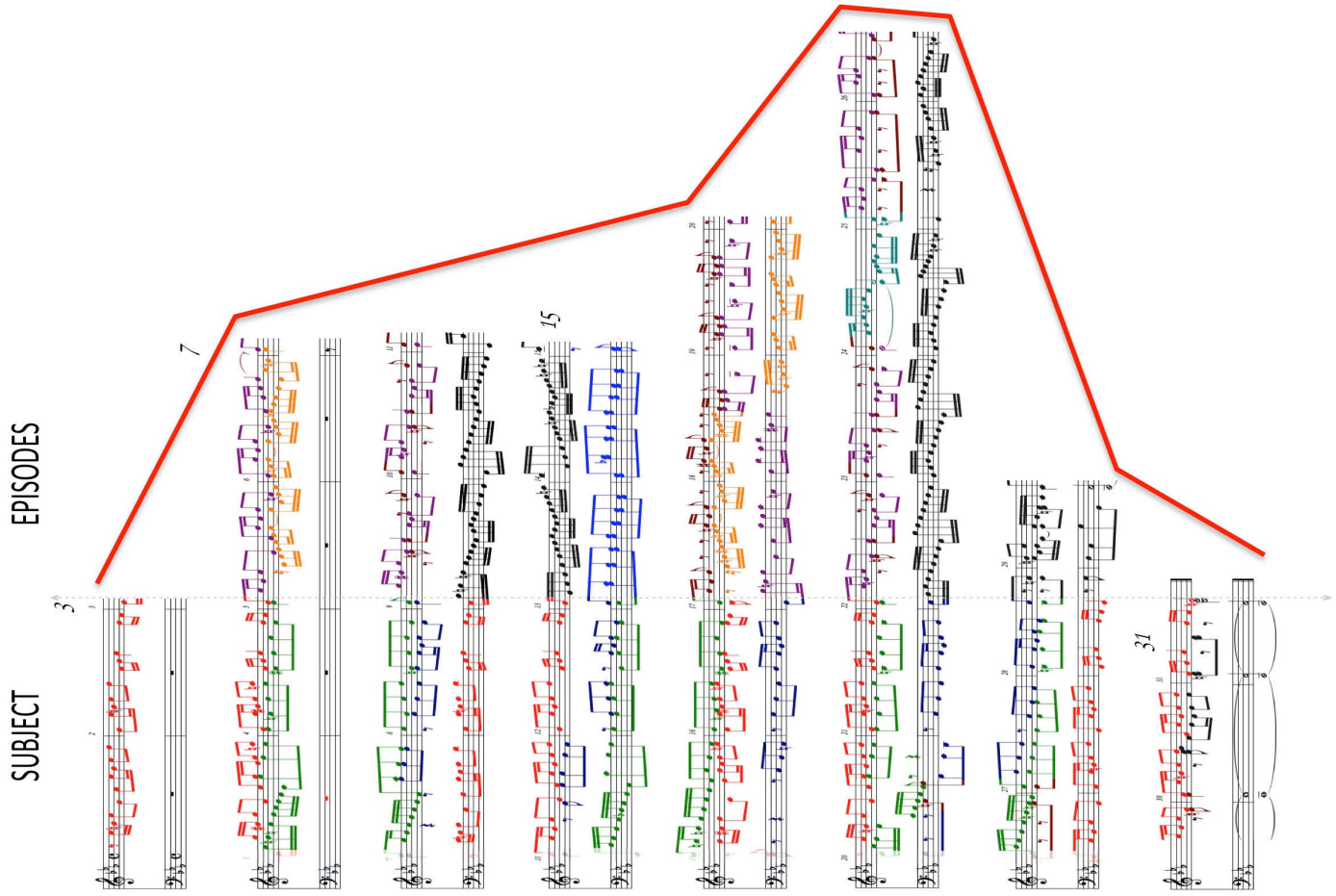
Example 13:



Example 13 analyzes the episodes by themselves. Please note that on the left (in black) we have the mathematical proportion and on the right (in red) measure numbers starting from one. The episodes make use of the rich materials from the subject and countersubjects while they build up tension and drama and explore new harmonies and

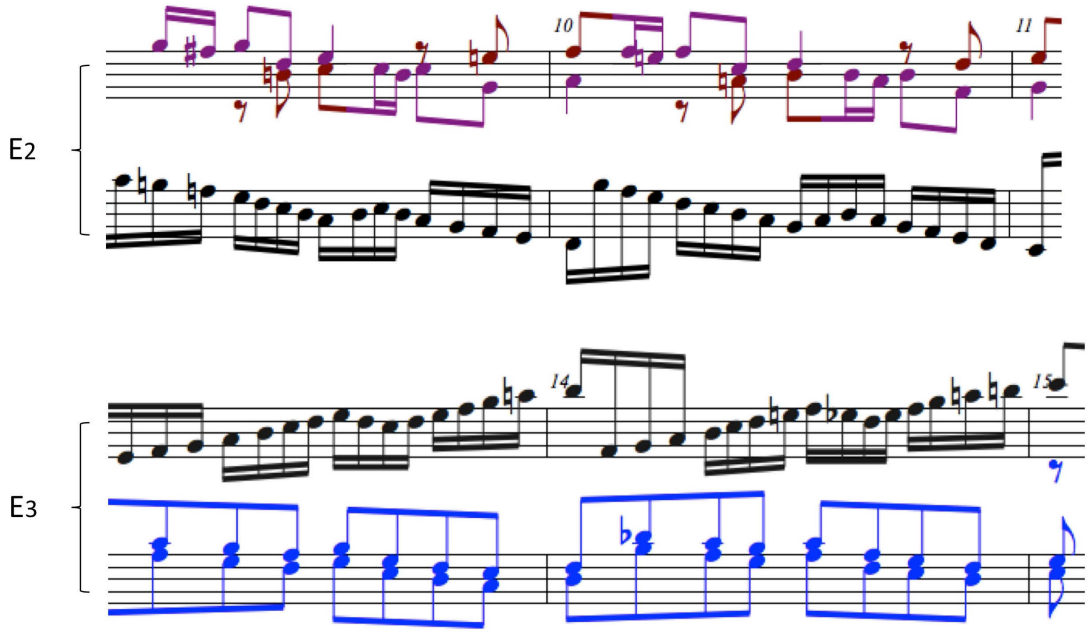
dissonances. By portraying music in this new Architectural Design, we notice that perhaps their most climactic effect lies in the extra long delay of the return of the subject as in measure 14 in example 13.

Example 14:

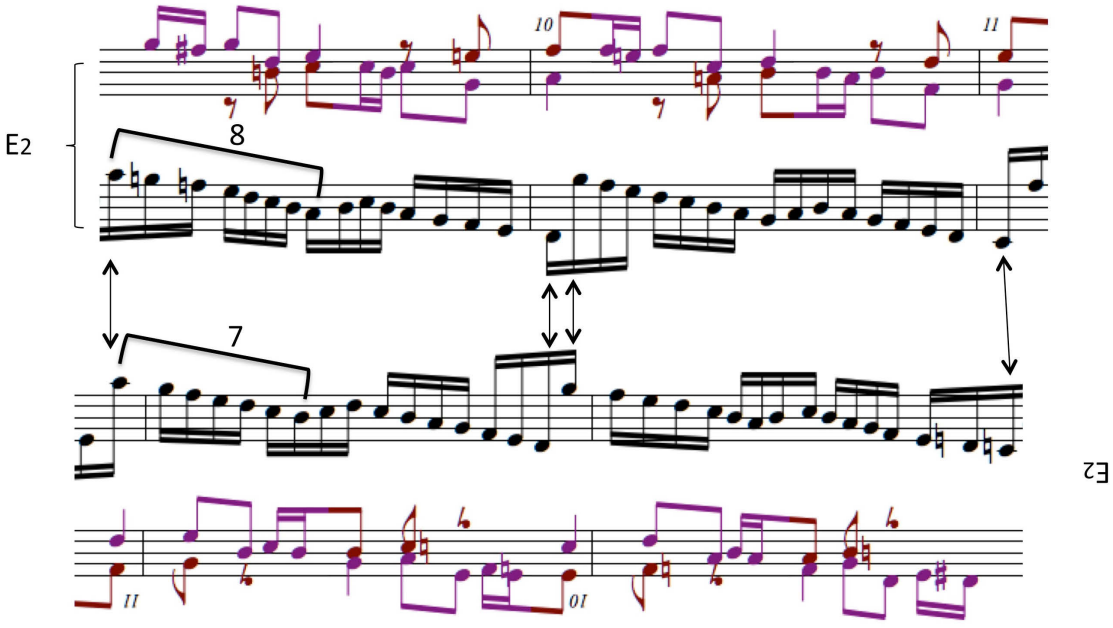


When we turn the work on its side as in example 14, we can clearly see the dramatic arch, as a skyline view where the higher you go the more tension in the piece, harmonically and dramatically.

Example 15:

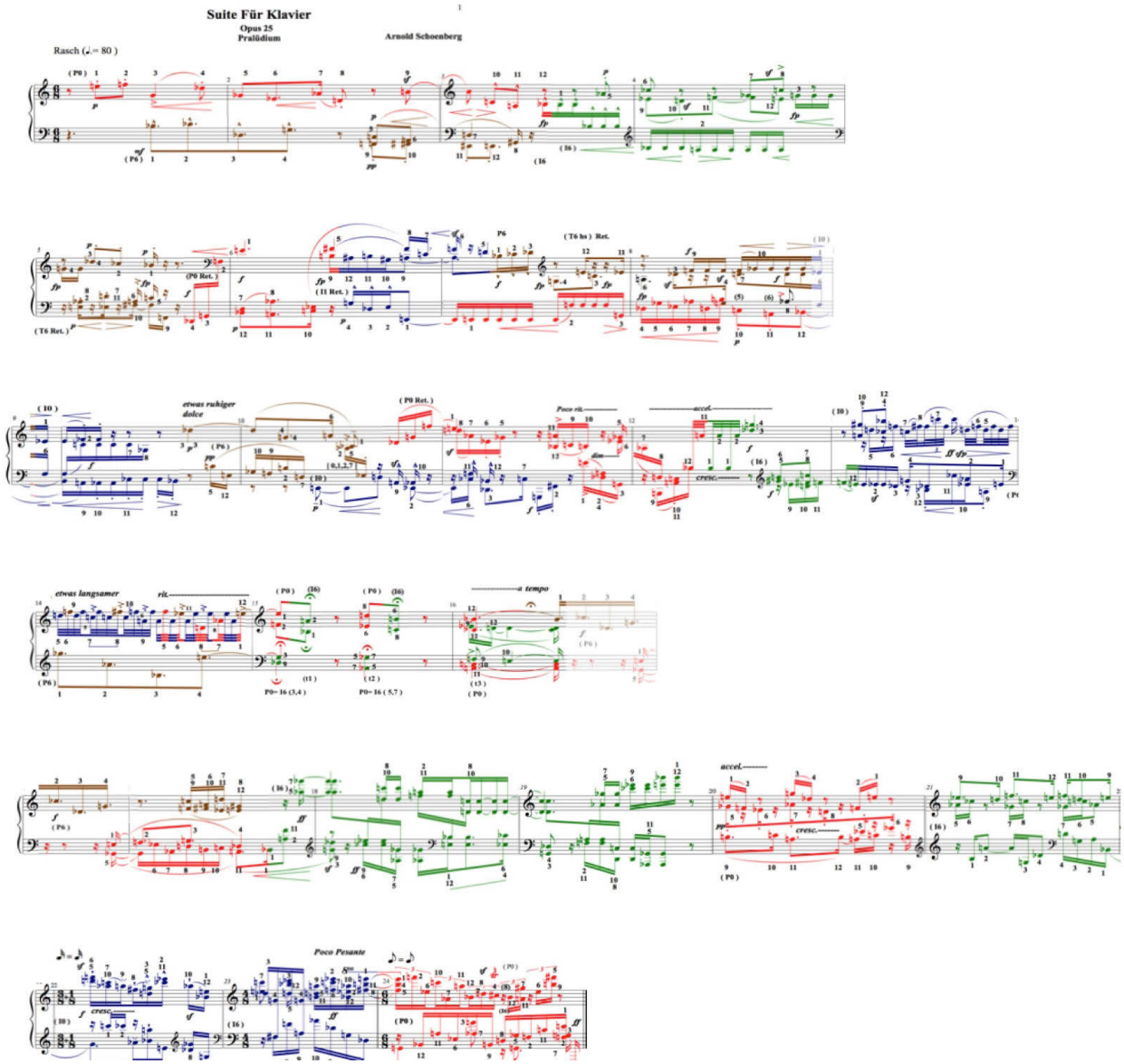


Example 15a:



When we compare episodes two and three (E2 and E3) in example 15, we can see with ease that the scales represented in black are a perfect inversion - a mirror image of itself. Furthermore, the mastery of Bach's command of music can be seen in example 15a; here we can see the second episode (E2) now upside down where a retrograde almost worked perfectly. Though a perfect retrograde did not work in this instance since it is one note short: 7 vs. 8 notes, the contour of high notes to low notes is perfect. All of these great details clearly come to light in this new layout as a reflection of the architectural design intended by the composer.

Example 16: Schoenberg's Piano Suite op. 25, Prelude



When we apply the same analytical method to the Prelude of Schoenberg's Suite op. 25 (example 16) we realize that although it is titled prelude, in reality it is a four voice fugal structure based on serial technique (12-tone). The beauty of the symmetrical mathematical underlying structure lies not only in the melodic and harmonic reciprocity expressed on Schoenberg's matrix but in the thematic mastery of palindromes, juxtapositions, inversions, retrograde, overlapping, interlocking and many more dodecaphonic techniques.

Example 17 contains the Matrix for the entire suite, which Schoenberg used as the base mathematical structure; however we will focus only on the prelude, which uses the outside edge of the Matrix only. Red in the score as well as the matrix represents the original set (P_0), the most important 12 note row on which the entire suite is based. In blue we find inversion zero (I_0), in brown transposition sum six (P_6) and in green inversion six (I_6); all of which Schoenberg exploits in retrograde, particularly at the tetrachord level.

In example 17f please notice the tritone polarity of the matrix – E to Bb – in which interval class six is used as the "Dominant" since it is the half way point in serialism.

Example 17:

	I_0	I_1	I_3	I_9	I_2	I_{11}	I_4	I_{10}	I_7	I_8	I_5	I_6	
P_0	E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb	R_0
P_{11}	Eb	E	F#	C	F	D	G	Db	Bb	B	Ab	A	R_{11}
P_9	Db	D	E	Bb	Eb	C	F	B	Ab	A	F#	G	R_9
P_3	G	Ab	Bb	E	A	F#	B	F	D	Eb	C	Db	R_3
P_{10}	D	Eb	F	B	E	Db	F#	C	A	Bb	G	Ab	R_{10}
P_1	F	F#	Ab	D	G	E	A	Eb	C	Db	Bb	Cb	R_1
P_8	C	Db	Eb	A	D	B	E	Bb	G	Ab	F	Gb	R_8
P_2	F#	G	A	Eb	Ab	F	Bb	E	Db	D	B	C	R_2
P_5	A	Bb	C	F#	B	Ab	Db	G	E	F	D	Eb	R_5
P_4	Ab	A	B	F	Bb	G	C	F#	Eb	E	Db	D	R_4
P_7	B	C	D	Ab	Db	Bb	Eb	A	F#	G	E	F	R_7
P_6	Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E	P_6
	RI0	RI1	RI3	RI9	RI2	RI11	RI4	RI10	RI7	RI8	RI5	RI6	

Example 17a:

E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
Eb											A
Db											G
G											Db
D											Ab
F											Cb
C											Gb
F#											C
A											Eb
Ab											D
B											F
Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E

Example 17b:

	1	2	3	4	5	6	7	8	9	10	11	12
Row P_0	E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
	Eb											A
	Db											G
	G											Db
	D											Ab
	F											Cb
	C											Gb
	F#											C
	A											Eb
	Ab											D
	B											F
	Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E

Example 17c:

Inversion

I_0

1	E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
2	Eb											A
3	Db											G
4	G											Db
5	D											Ab
6	F											Cb
7	C											Gb
8	F#											C
9	A											Eb
10	Ab											D
11	B											F
12	Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E

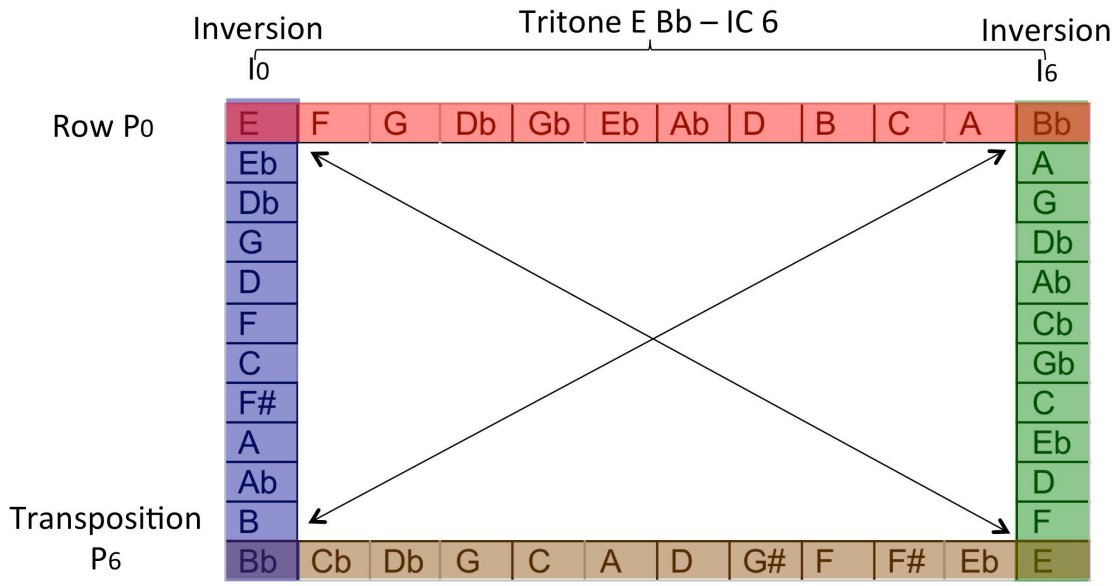
Example 17d:

	E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
	Eb											A
	Db											G
	G											Db
	D											Ab
	F											Cb
	C											Gb
	F#											C
	A											Eb
	Ab											D
Transposition	B											F
P_6	Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E
	1	2	3	4	5	6	7	8	9	10	11	12

Example 17e:

											Inversion I_6	
E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb	1
Eb											A	2
Db											G	3
G											Db	4
D											Ab	5
F											Cb	6
C											Gb	7
F#											C	8
A											Eb	9
Ab											D	10
B											F	11
Bb	Cb	Db	G	C	A	D	G#	F	F#	Eb	E	12

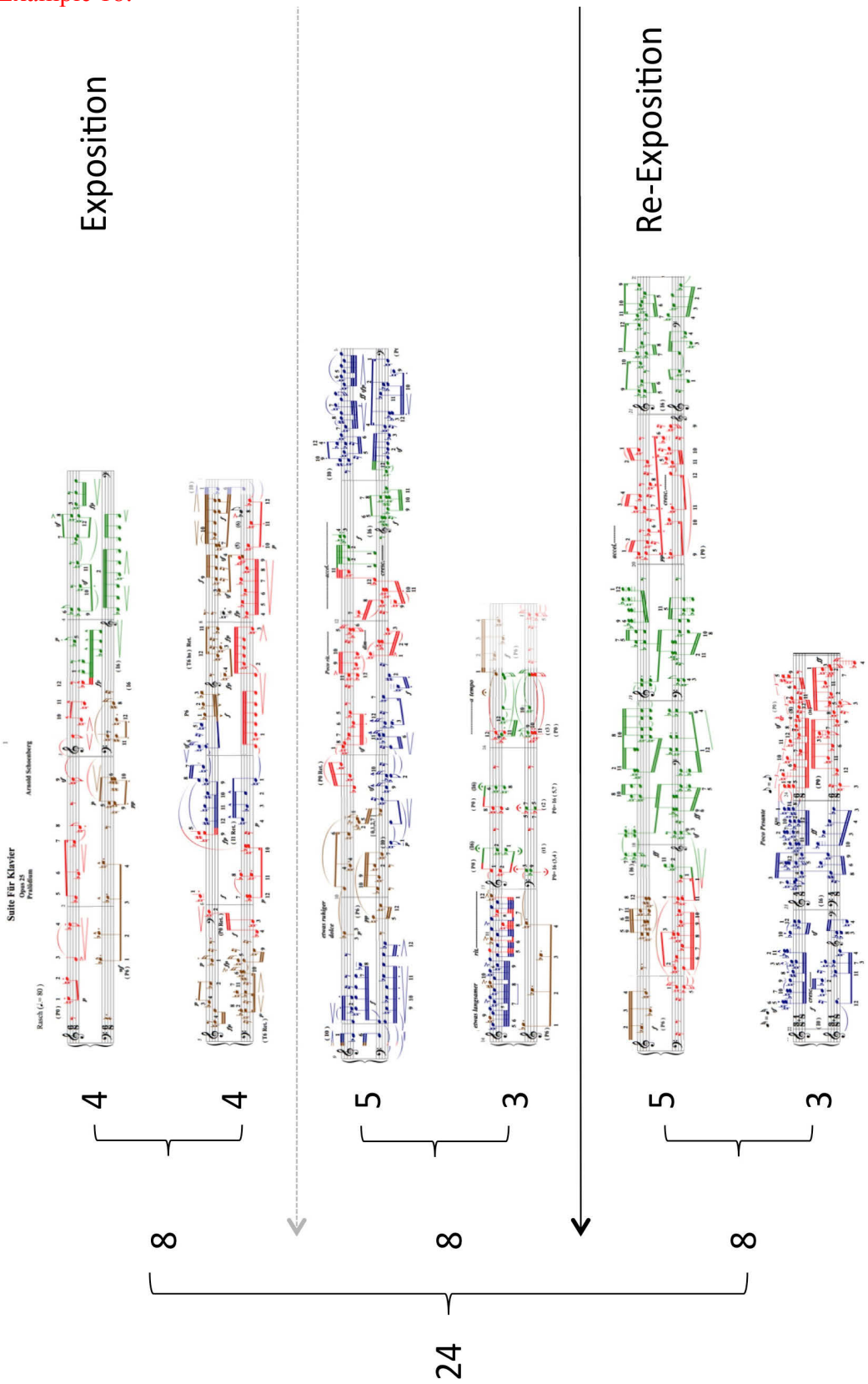
Example 17f:



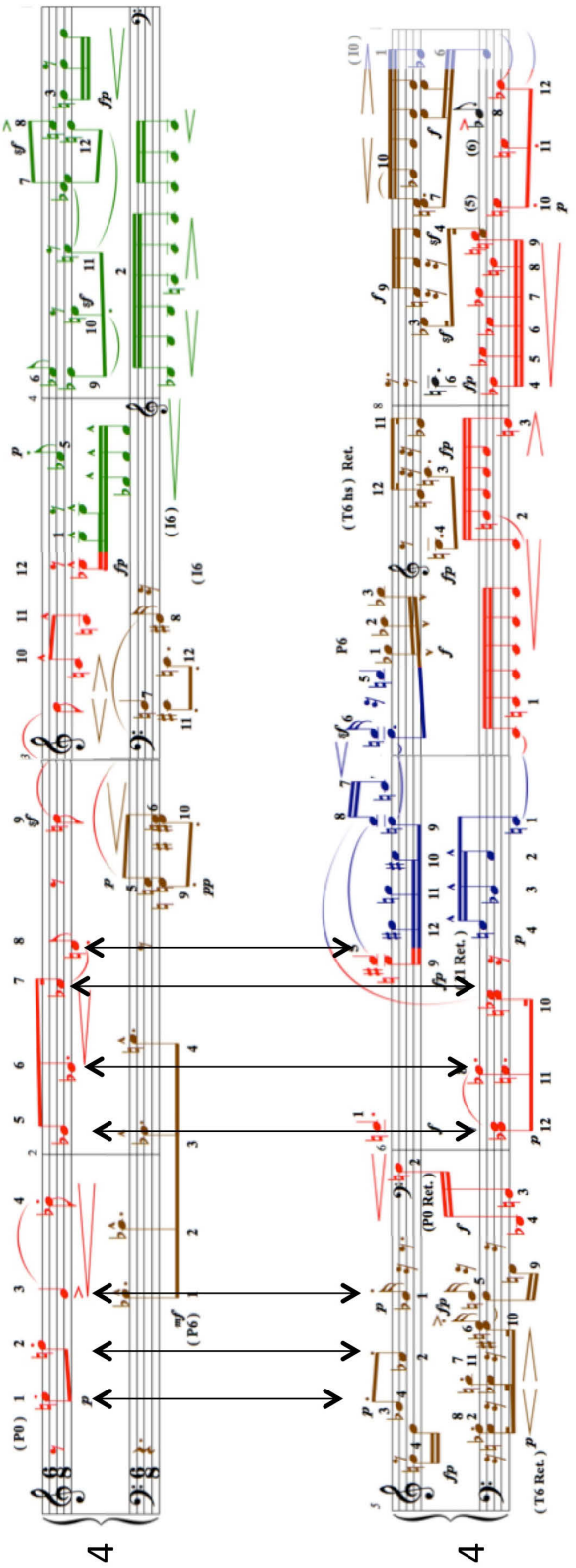
Regarding the form and architectural design of this prelude, in example 18 we have three groups or sections made up of eight bar phrases where the first section is divided equally in half (4 + 4) and the second and third sections in Golden proportion (5 + 3). Thematically speaking, from a fugal perspective, we have a clear exposition and re-exposition treated in a very modernist way. A relevant question remains: Why this particular layout? Why divide the work like this since clearly there is not a melodic restatement of the main subject P_0 (red) as in the example by Bach?

In example 19 we find the answer: rhythm takes precedence over melody. The rhythmic restatement of the subject in the second system is a transformation, presented in a retrograde inversion at the tetrachord level (T_6) shown in brown.

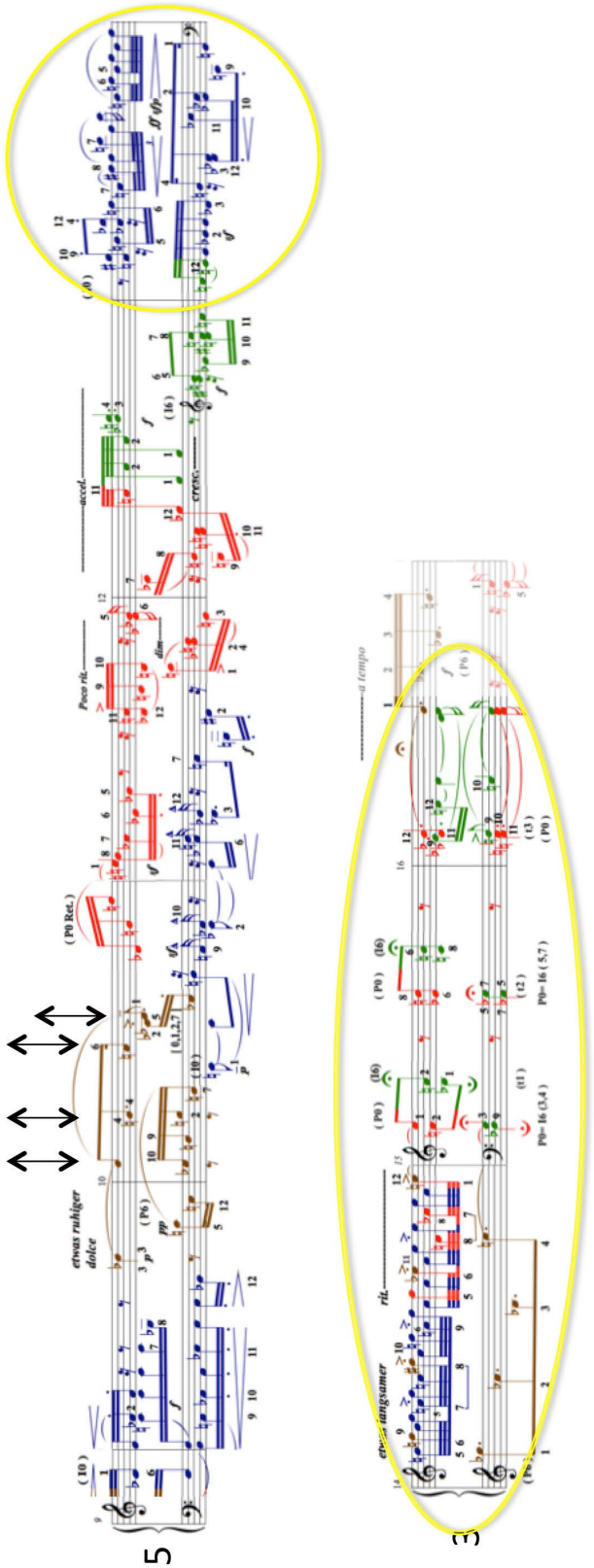
Example 18:



Example 19:



Example 20



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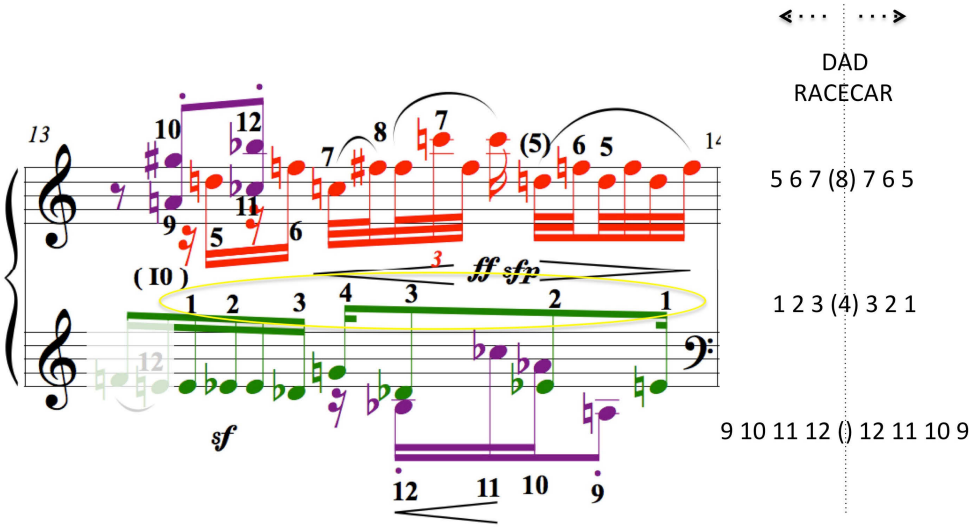
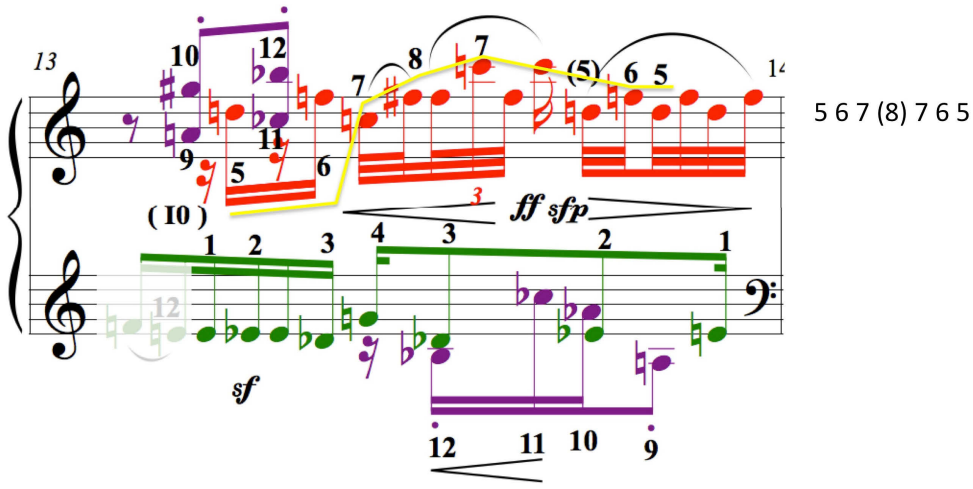
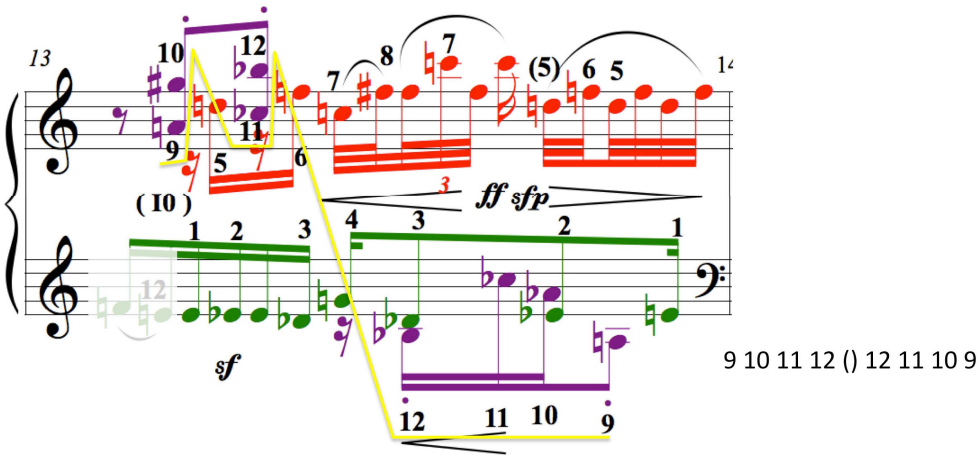


Example 22:

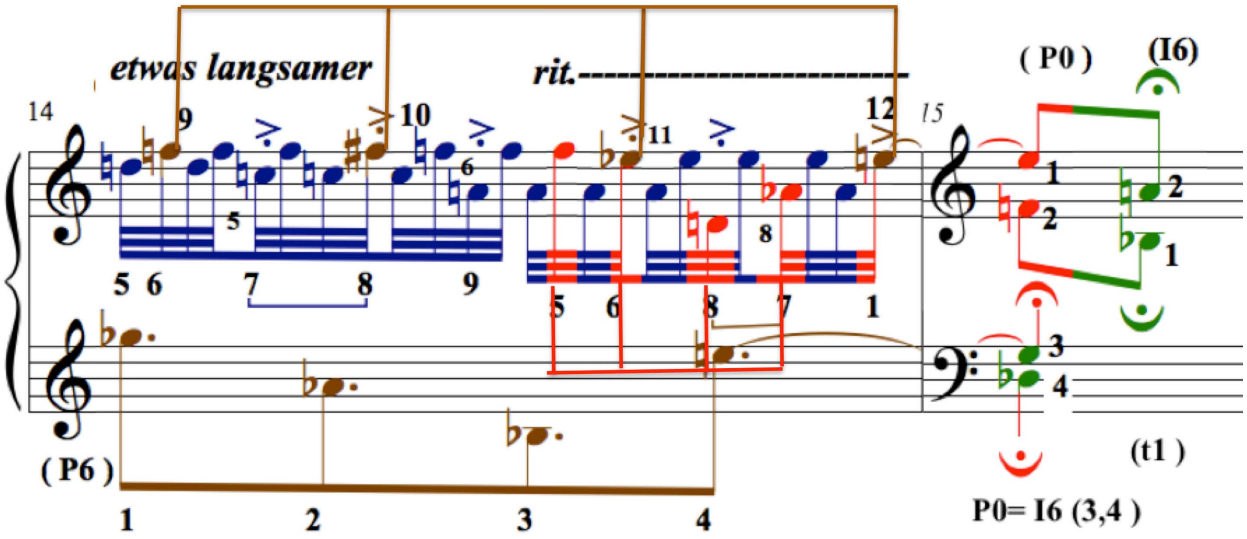


Example 23:



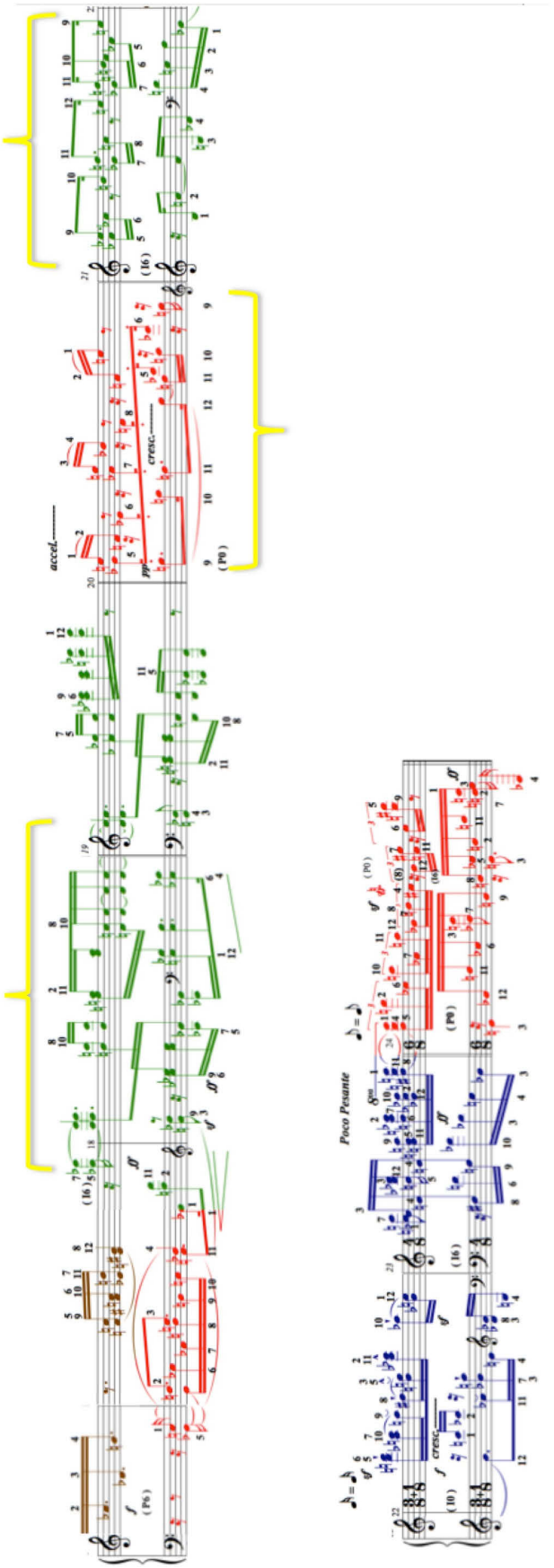
In the second section (example 20) Schoenberg utilizes two new techniques, the use of palindromes in measure 13 and voice overlapping in measure 14. Looking at measure 13 in detail (example 21) we find three overlapping palindromes, the easiest to see is represented in green as 1 2 3 4 3 2 1, followed by (example 22) 5 6 7 8 7 6 5 in red and (example 23) 9 10 11 12 12 11 10 9 in purple. The overlapping technique in measure 14 (example 24) at the tetrachord level is an ingenious use of accents and common notes by Schoenberg.

Example 24:



Section three acts as the re-exposition (example 25) which deals with palindromes and complete presentations of each voice in this fugal structure. The second system represents a three-measure long coda focusing on dyads and trichords. Examples 26-29 display a new mathematical underlying in which four trichords of the set are proportionally built using the Clock Wheel. The first trichord 1 9 5 (example 26), the second trichord 2 6 10 (example 27), the third 7 11 3 (example 28) and the fourth 12 8 4 (example 29)

Example 25:

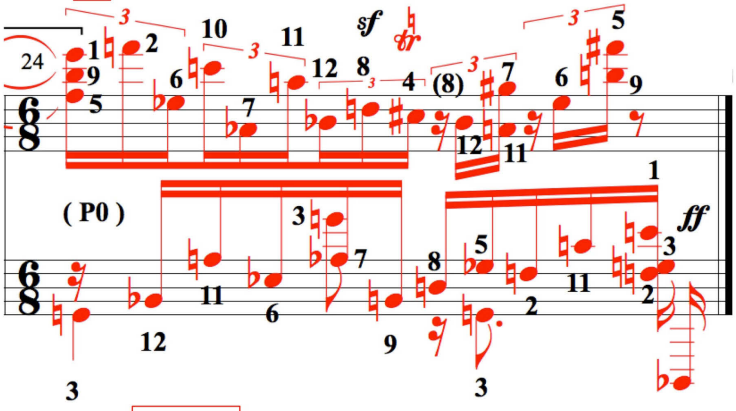


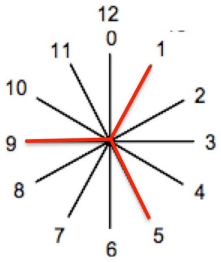
Trichords

Dyads-	1-12,	2-11,	3-4,	5-7,	6-9,	8-10
IC	6	4	6	2	4	2

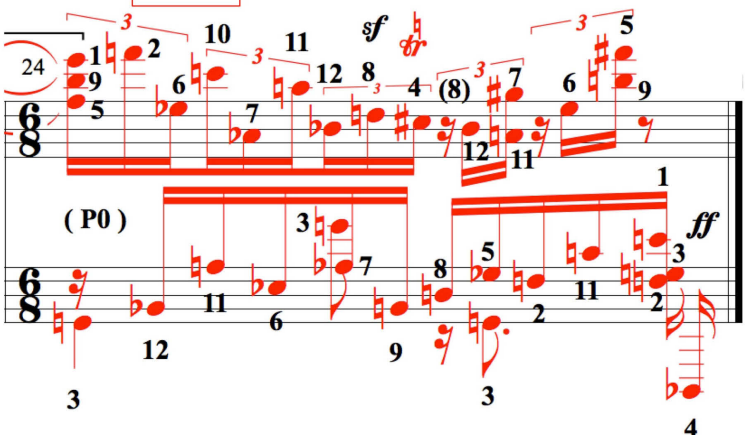
Examples 26-29:

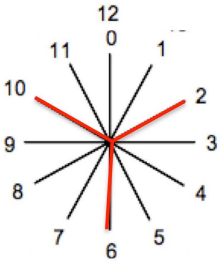
1 9 5



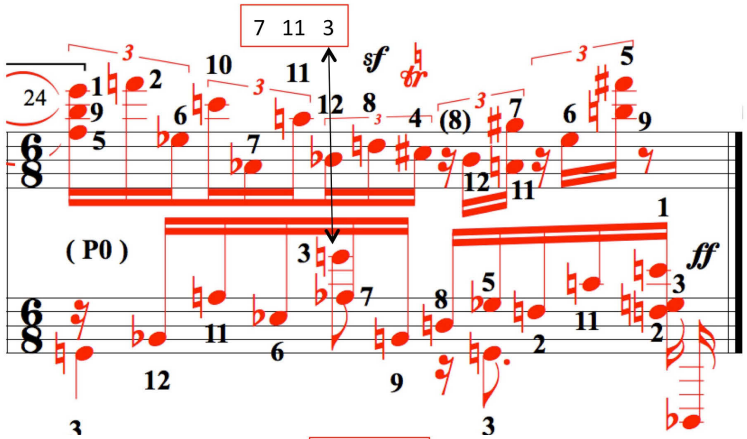


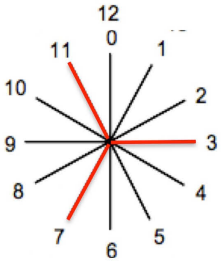
2 6 10



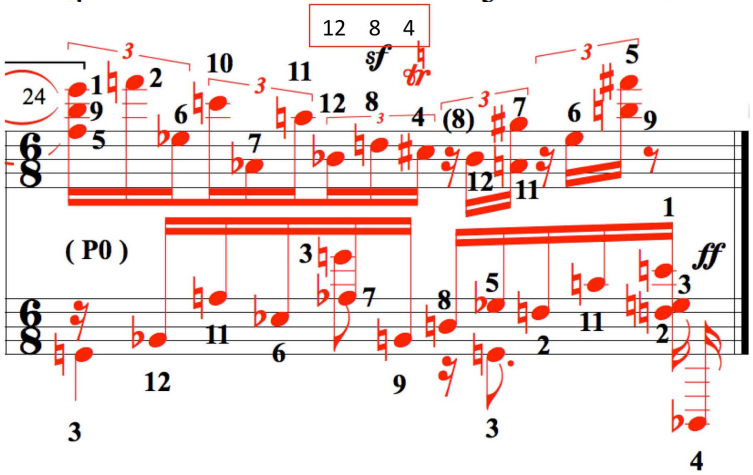


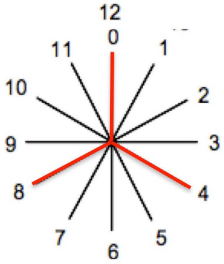
7 11 3



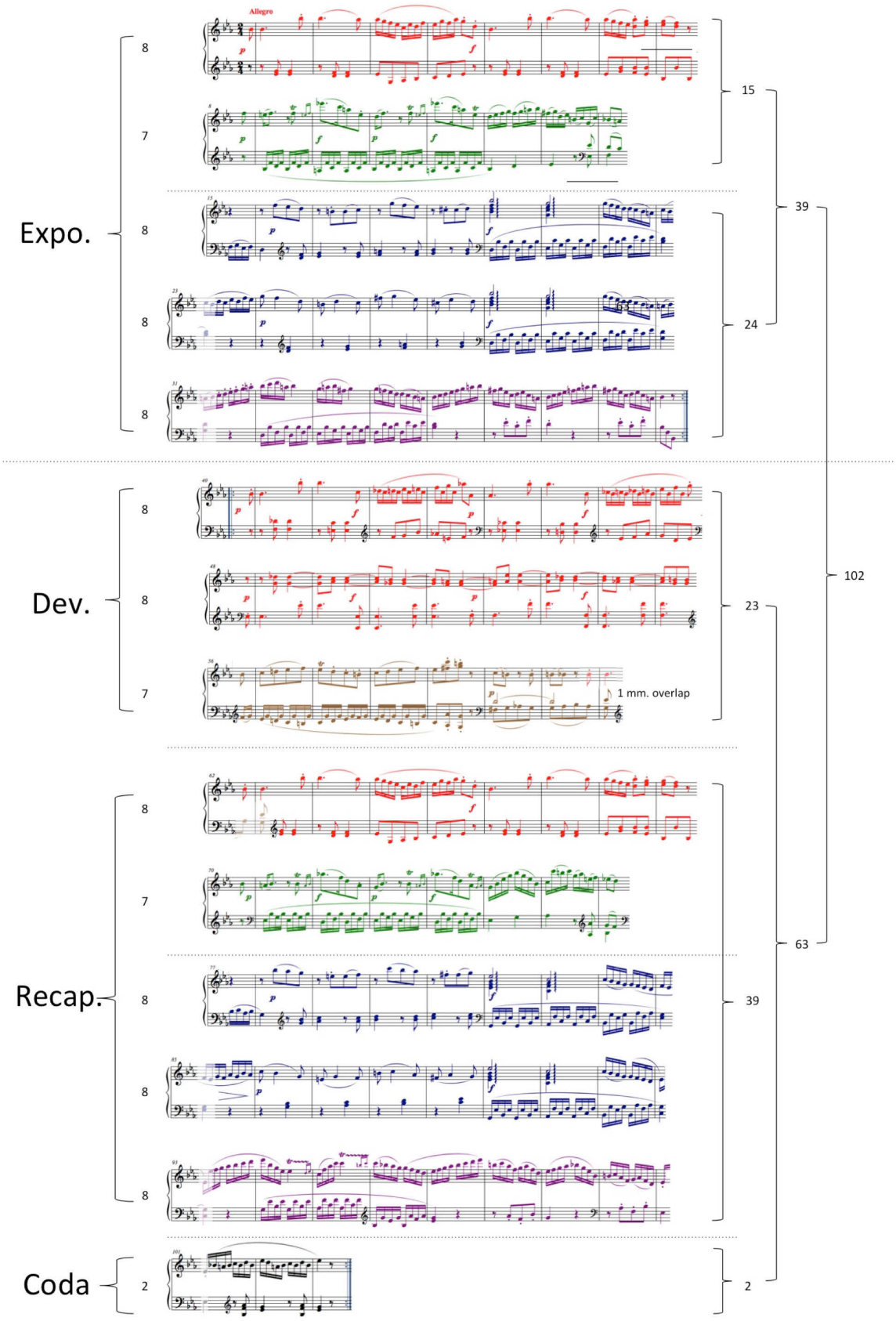


12 8 4





Example 30: Mozart's Piano Sonata No. 4 in E-flat major, K. 282. III



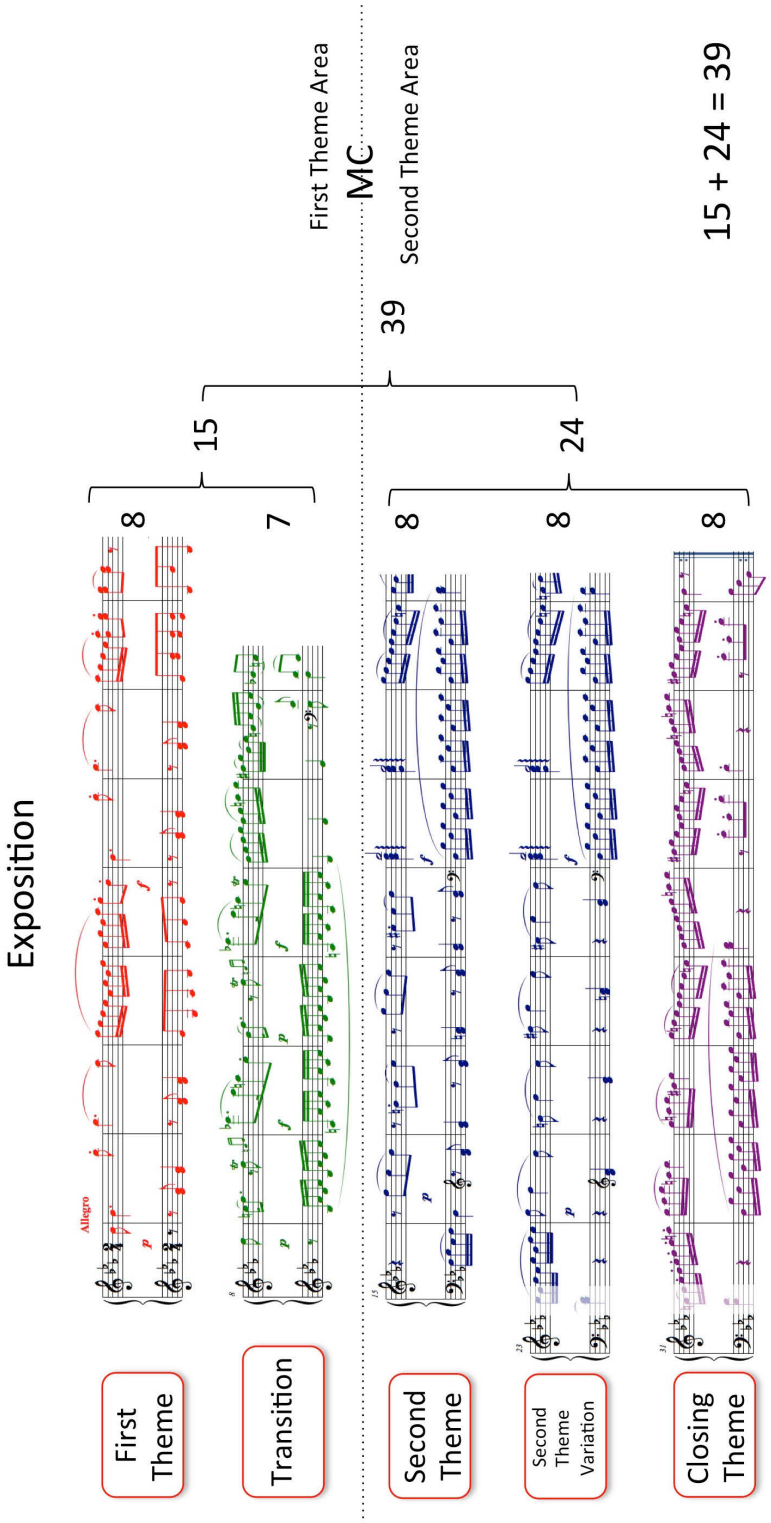
Lastly, let us apply this method to sonata allegro form. Example 30 displays the entire third movement (102 measures long) of Mozart's Piano Sonata No. 4 in E-flat major, K. 282. Please keep in mind that everything is aligned and that every phrase (verse/system) is 8 measures long except for the three instances of the 7 measure long phrase and the 2 measure coda (codetta). The author has searched high and low for the meaning of those 7 bar phrases and has not found in any book a satisfying explanation for them. They occur everywhere in Mozart, Haydn, Beethoven, Schubert and are always present in sonata form without exception.

The answer to the riddle of the seven bar phrase lies not only in the overall design of the piece, but also in the small details of each section: "One thing arises from all things, and all things arise from one thing" – Heraclitus. In the exposition (example 31) the 7 measure long phrase (the transition) occurs in order to allow the large-scale architectural design of the piece to exist. This irregular phrase is not arbitrarily created, but is part of the greater poetic and formal scheme of the work. Please note that the dotted line represents the medial caesura that separates the first theme area from the second.

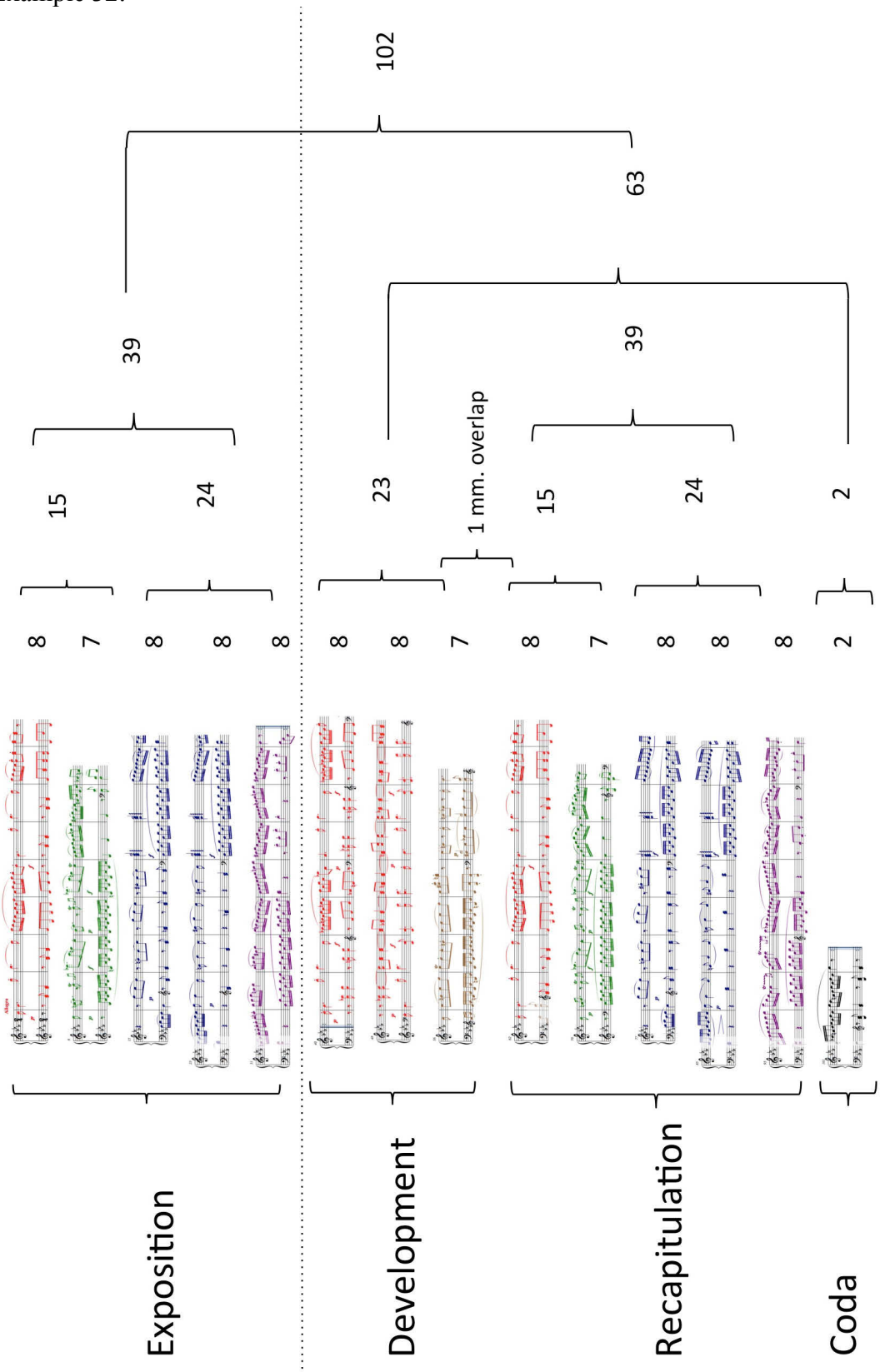
In example 32, a clear mathematical proportion emerges to clarify the division of themes and phrases. The first theme area is 15 measures long and the second theme area is 24 measures long; adding 15 + 24 = 39 and 39 (expo) + 63 (development/recap/coda) = 102. At this point we can no longer avoid the presence of the Golden Proportion, which is perhaps the most optimal way of combining binary and ternary form. The numbers that provide the best approximation to the Golden Ratio can be examined in example 33. The

Golden Ratio in musical terms states that the exposition is to the development/recap/coda as the development/recap/coda is to the entire movement (the total sum of the exposition/development/recap/coda).

Example 31:

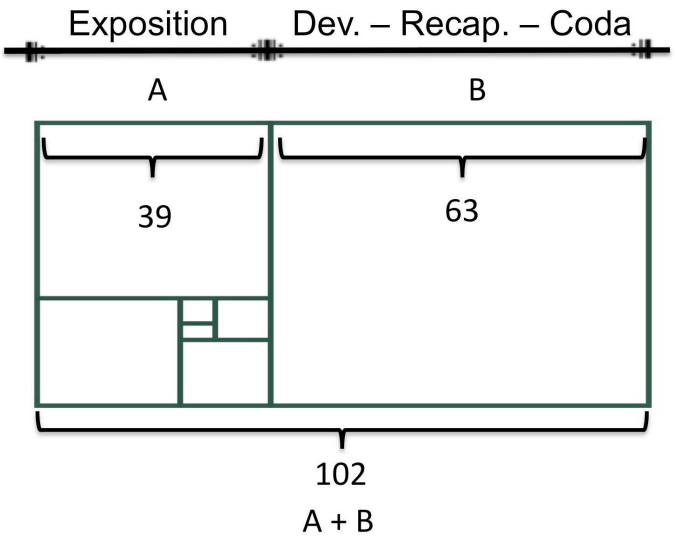


Example 32:



Example 33:

Golden Ratio $\varphi = 1.618...$



A is to B as B is to A + B

$$\frac{A}{B} = \frac{B}{A+B} = \frac{1}{\varphi} = \varphi - 1 \approx 0.618$$

Sequence: 39 63 102

$$\frac{102}{63} \approx 1.619 \qquad \frac{63}{39} \approx 1.615$$

Once we organize music in a coherent phrase structure, the form reveals itself. In addition to presenting a practical and holistic method of portraying a score, The Illuminated Scores provide a deeper structural understanding of the composition, which makes the themes and underpinnings obvious to the performer. What used to take years or decades of analysis can now be more easily seen from the onset. This new platform could become indispensable for performers and educators, as the form reveals itself. This leaves one excited about the possibility of exploring the fascinating tools employed by the most brilliant minds in the world of music.

Music as an artistic expression is able to communicate the relationship between poetry, architecture and musical design with ease, and mathematics helps us prove these relationships, while clarifying the meaning of seemingly unexplainable musical ideas such as the meaning of irregular phrases and intricate overlapping techniques.

The author encourages all to explore this new perspective for its cognitive advantages, especially in regards to visual memory and a deeper understanding of the meaning of music. The Illuminated Scores from a visual perspective, serve as a roadmap to understanding the meaning of "The Architectural Design of Musical Form."